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301304

December, 2019

B.Tech. (CE/IT/CSE) 3rd SEMESTER

Mathematics-III (Calculus and Ordinary Differential Equations) (BSC-301)

Time : 3 Hours] [Max. Marks : 75

Instructions :

1. *It is compulsory to answer all the questions (1.5 marks each) of Part -A in short.*
2. *Answer any four questions from Part -B in detail.*
3. *Different sub-parts of a question are to be attempted adjacent to each other.*

PART-A

1. (a) Discuss the convergence of the series : $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$.

(b) Check the convergence of $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\ldots\ldots$

(c) If $u = (x^2 + y^2 + z^2)^{-1/2}$, then prove that

$$\frac{\partial^2 u}{\partial^2 x}+\frac{\partial^2 u}{\partial^2 y}+\frac{\partial^2 u}{\partial^2 z}=0.$$

(d) Find the equation of the tangent plane and normal to the surface of $2xz^2 - 3xy - 4x = 7$ at $(1, -1, 2)$.

(e) Solve $y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)dy = 0$.

(f) Solve $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$. (solve for p).

(g) Evaluate $\int_0^5 \int_0^{x^2} x(x^2 + y^2)dydx$.

(h) Solve $\int_0^1 \int_0^2 \int_1^2 x^2yzdxdydz$

(i) Find the P.I. of $(D^2 + 5D + 6)y = e^x$.

(j) Solve $(D^4 + 4)y = 0$. (1.5×10=15)

PART-B

2. (a) Test for the convergence of the series

$$\sum \frac{4.7.....(3n+1)}{1.2.....n} x^n .$$ (8)

(b) State, with reasons, the values of x for which the series :

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - ..... \text{ converges .}$$ (7)

3. (a) Prove that the rectangular solid of maximum volume which can be inscribed in a sphere is a cube. (8)

(b) Show that $\nabla^2(r^n) = n(n+1)r^{n+2}$. (7)

4. (a) Evaluate $\iint \frac{rdrd\theta}{\sqrt{a^2 + r^2}}$ over one loop of the lemniscates $r^2 = \cos 2\theta$. (8)

(b) Apply Stoke's theorem to evaluate $\int_c (ydx + zdy + xdz)$, where c is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$. (7)

5. (a) Solve $(1 + y^2)dx = (\tan^{-1}y - x)dy$. (8)

(b) Solve $y = 2px + p^n$. (7)

6. (a) Solve by the method of variation of parameters, $(D^2 - 2D + 1)y = e^x \log x$. (8)

(b) Derive an expression for the orthogonality property of the Legender's polynomial. (7)

7. (a) Express the vector field $2y\mathrm{I} - z\mathrm{J} + 3x\mathrm{K}$ in spherical polar coordinate system. (8)

(b) Prove that $d/dx[xJ_n(x)J_{n+1}(x)] = x[J^2_n(x) - J^2_{n+1}(x)]$. (7)